Calculation of Atmospheric Emission Measured by Gregorian Feed Horns

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As we use higher frequencies, emission from the Earth's atmosphere becomes more significant. This is important not only in determining system temperatures, but for initial receiver calibration of the receiver temperatures and noise diodes.

The usual procedure is to take the receiver looking vertically in the receiver test laboratory. Calibration is effected by measuring output when the input is looking at the 'cold' sky and at an 'ambient load'. The ambient load is assumed to be at the local temperature, but the question is what temperature to assign for the sky.

The atmospheric temperature is due to the low-frequency wings of molecular lines, primarily those of H_2O, but to a lesser extent of O_2. The exact shape of these line wings is hard to model, but there are standard programs that do this. One such is included in the set of routines called "ASTRO" in the package "CLASS", and is called "ATM". To use this program, you input the altitude, temperature, and precipitable water vapor (pwv) in mm. The latter is the most variable quantity–it is the integral of the water content of the atmosphere from starting altitude on up.

Luca Olmi ran an atmospheric model code, and generated the following values for pwv:

T(C)	RH(%)	pwv(mm)
28	80	44.0
25	70	32.7
30	85	52.1

Thus, we can expect pwv's between 30 and 50 mm at Arecibo. These are, not surprisingly, considered large values. I have run the ATM code for several values of pwv with plausible temperature, and generated the zenith sky temperature as a function of frequency between 1 and 15 GHz. I have added the Cosmic Background Radiation (CBR) set to be 2.74 K. No other Galactic emission is included.

The sky temperature at 5 GHz is almost unaffected by the water content of the atmosphere, and is close to 5 K under all conditions. At 10 GHz the situation is somewhat different, and we find the results given in the following table:

pwv(mm)	T_{sky}(10GHz) @ zenith (K)
10	6.0
30	7.3
50	9.0

Thus, the zenith temperature could be vary between 6 K under exceptionally dry conditions, and perhaps reach as high as 10K, under very humid conditions. These results do not explicitly include the effect of clouds or precipitation.

One further question is the *effective sky temperature* resulting from the feed horn's radiation pattern–it is not sensitive only to radiation from the zenith, but has a fairly wide acceptance pattern. The standard for Gregorian horns is to be down about 15 dB (relative sensitivity = 0.03) at 60 degrees from boresight (in this case, the zenith).

A reasonable way to describe the main lobe of the radiation pattern of broad angle feed horns, neglecting their sidelobes and other peculiarities, is as a $cos^n(\theta)$ distribution, where θ is the angle off–boresight. This is only applicable for angles between 0 degrees and 90 degrees. The exponent n is determined by the relative power P_{rel} at a specific angle θ_{rel} through the relationship

$$n = \frac{log(P_{rel})}{log(cos\theta_{rel})} \ . \tag{1}$$

From our specification of P_{rel} = 0.03 for θ_{rel} = 60 degrees, we find that n = 5.06.

The antenna temperature measured is given by the convolution of the sky temperature and the feed horn response pattern through

$$T_A = \frac{1}{\Omega_A}\int\int T(\Omega)P_n(\Omega)d\Omega \ . \tag{2}$$

For the present azimuthally symmetric situation, assuming that the atmosphere is optically thin, we have

$$T(\Omega) = T(\theta) = T_0/cos\theta \ , \tag{3}$$

where T_0 is the atmospheric temperature at zenith. Putting in the expression for the feed horn response, and using limits on the integral of 0 degrees and 90 degrees, we find

$$T_A = \frac{2\pi T_0}{\Omega_A}\int cos^{n-1}\theta sin\theta d\theta = \frac{2\pi T_0}{n\Omega_a} \ . \tag{4}$$

The expression for the antenna solid angle is very similar:

$$\Omega_A = \int\int P_n(\Omega)d\Omega = 2\pi\int cos^n\theta sin\theta d\theta = \frac{2\pi}{n+1} \ . \tag{5}$$

Combining these we obtain the result that

$$T_A = T_0\frac{n+1}{n} \ . \tag{6}$$

The result is actually not very sensitive to the tertiary edge taper (value of P_{rel}, varying from a factor of 1.30T_0 for P_{rel} = -10 dB, to 1.20 for our standard value P_{rel} = -15 dB, to 1.12 for P_{rel} = -25 dB. Thus, although the beam width of the feed horn certainly does have an effect on the sky temperature measured, it is not a big contributor.

Using the standard value and associated factor $T_A = 1.2T_0$, we obtain the following values (note that you have to subtract the CBR values from previous table entries as they are *not* scaled by factor 1.2, and then add them back in):

pwv(mm)	T_A(10GHz) @ zenith (K)
10	6.7
30	8.2
50	10.3

I think these values should be reasonably accurate, but I have not carried out any specific tests of the atmospheric emission code. I would appreciate any comments on this, and on the feed horn modeling as well. However, note also that we are not including large–angle feed horn sidelobes, which can add in ground pickup. That is one reason why a "sky cone" which I discussed with Edgar Castro seems like a good bet in almost any case.