To: Bill Sisk file: s_param_2port
From: Jon Hagen
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Subject: Reciprocal 2-port junctions

If a network is *reciprocal*, then $S_{ij} = S_{ji}$. 1) (2 equations - real part and imag. part)

leaving three complex parameters, S_{11}, S_{12}, and S_{22} (six real parameters).

I. Lossless Case

For a lossless 2-port $|S_{11}|^2 + |S_{21}|^2 = 1$ 2) (1 equation, from conservation of energy when only Port 1 is excited)

$|S_{22}|^2 + |S_{21}|^2 = 1$ 3) (1 equation, from conservation of energy when only Port 2 is excited)

These two equations reduce the six real parameters to four real parameters. A final relation is

$S_{11}\, S_{12}^* + S_{21}\, S_{22}^* = 0$ 4) (from conservation of energy, when both ports are excited).

This appears to be two equations (real and imaginary parts), but actually amounts to only one equation. To see this (reference: R.E. Collin, Foundations for microwave engineering) , note first that Equations 2 and 3 imply that $|S_{11}| = |S_{22}|$. We can therefore express the S parameters in polar form as $S_{11} = |S_{11}|\, e^{-j\theta 1}$, $S_{22} = |S_{11}|\, e^{-j\theta 2}$, and $S_{21} = (1- |S_{11}|)^{½}\, e^{-j\varphi}$. Putting these expressions into Equation 4 gives

$$(|S_{11}|)(1- |S_{11}|)^{½} [\, e^{-j(\theta 1-\varphi)} + e^{-j(\varphi-\theta 2)}] = 0 \qquad 5)$$

The term in the square brackets must be zero, i.e.

$$(\theta 1-\varphi) = (\varphi-\theta 2) + \pi + 2n\, \pi \quad \text{where n is a pos. or neg. integer} \qquad 6)$$

Thus Equation 4 boils down to a single equation and the four real parameters are finally reduced to just **three real parameters**. Any lossless 2-port can therefore be modeled (for one frequency) as a network made of **three elements**, which can be capacitors, inductors, and transmission lines. As an example of such a network, suppose we start with the capacitor, which, when paralleled with 50 ohms of resistance, produces an impedance Z, such that $|(1-Z)/(1+Z)| = |S_{11}|$. Let this capacitor be a shunt element to ground. To its left, install a cable with the length needed to get the correct phase for S_{11}. To its right, install a cable whose length produces the specified phase for S_{22}.