Radio Astronomy Techniques

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    Minimum noise measure over a given bandwidth - a "Fano's theorem" for noise

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    measure small insertion loss of transmission devices
        via high power transmission & heating
        via noise measurement
        theoretical, e.g. HFSS to get good picture of current distribution

    simple theory vs. complicated consideration of all mismatches
    References: NRAO Electronics Div. Report No. 143, Craig Moore & Chas. Brockway, May 1974

    noise measure

3. Baselines
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Standing Waves and Baselines

When mismatches exist at both ends of any connecting cable or waveguide in a receiver chain, ripples appear in the passband. The period of the ripple is given by $\frac{1}{2}c/L$ where c is the phase velocity in the cable. A 10 foot length of cable with a velocity factor of 0.7 would produce ripples with a period of 49.2 MHz. For small ripples, the size of the ripple is given by

$$[\text{Max. power/Min. power}] \text{ in dB} = 40 \log(e)\, |\rho_1|\, |\rho_2| = 17.4\, |\rho_1|\, |\rho_2|$$

where $|\rho_1|$ and $|\rho_2|$ are the magnitudes (not in dB) of the voltage reflection coefficients seen at the ends of the cable (source and load). For example, if $|\rho_1|$ and $|\rho_2|$ are both equal to 1/10 ("20 dB return loss"), the ripple is 17.4(1/10)(1/10) = .174 dB (about 4%). The ripple can be reduced by adding an attenuator. If a 10dB attenuator, for example, is placed at one end of the line, the reflection coefficient at that end of the line is reduced by $(1/\sqrt{10})^2 = 1/10$ due to the round trip through the attenuator. If the ripple had been 0.174 dB, it will be reduced to 0.174dB /10 = 0.0174 dB. Note that the attenuator can be placed anywhere along the line.

These conclusions follow from analyzing a circuit consisting of a sine wave generator, a transmission line, and a load. The generator is taken to have a frequency ω, an open-circuit output of 1 Volt peak, and an impedance Z_1. The transmission line has a characteristic impedance Z_0 and a length L. The load impedance is Z_2. The analysis shows the voltage at the load is given by

$$V_2(\omega)_{peak} = \left| \frac{2 z_2\, (1+z_1)^{-1} (1+z_2)^{-1}}{1 - e^{-j\omega 2L/c}\, \rho_1 \rho_2} \right|$$

where $z_1 = Z_1 / Z_0$, $z_2 = Z_2 / Z_0$, and the reflection coefficients are given by $\rho_1 = (z_1 - 1)/(z_1 + 1)$ and $\rho_2 = (z_2 - 1)/(z_2 + 1)$. If we assume the only significant frequency dependence over the band of interest is contained in the $e^{-j\omega 2L/c}$ term, the ripple voltage ratio is given by

$$\frac{V_{2\,\max}}{V_{2\,\min}} = \frac{1 + |\rho_1 \rho_2|}{1 - |\rho_1 \rho_2|}.$$

The power ripple in dB can therefore be written as

$$[Max.pwr/Min.pwr]in\ \ dB = 20\log(1+|\rho_1\rho_2|) - 20\log(1-|\rho_1\rho_2|)$$

$$= 20\log(e)[\ln(1+|\rho_1\rho_2|) - \ln(1-|\rho_1\rho_2|)].$$

For $|\rho_1|\,|\rho_2| \ll 1$ this reduces to $40 \log(e)\, |\rho_1|\, |\rho_2| = 17.4\, |\rho_1||\rho_2|$.

The same expression for the ripple can also be obtained using a time domain analysis. The signal at the output end of the cable consists of the original signal plus a series of echos caused by reflections at the output end and re-reflections from the input end. For simplicity, we will assume that the reflection coefficients are small, allowing us to ignore all but the first echo. In this case, the signal at the output end is given by

$$V_2\,(t) = V_1\,(t) + \rho_1\,\rho_2\,V_1\,(t\text{-}2L/c).$$

Again, for simplicity, we have assumed that the reflection coefficients are real. The autocorrelation function, $R(\tau) \equiv \langle V_2\,(t)V_2\,(t\text{-}\tau)\rangle$, of this voltage is easily shown to be

$$R(\tau) = R_0(\tau) + \rho_1\,\rho_2\,[\,R_0(\tau+2L/c) + R_0(\tau\text{-}2L/c)]$$

where $R_0(\tau)$ is the autocorrelation function of the input voltage and where we have discarded the $(\rho_1\,\rho_2)^2$ term. The power spectrum is just the Fourier transform of the autocorrelation function:

$$S(\omega) = \int R(\tau)e^{-i\omega\tau}d\tau = S_0(\omega)+\rho_1\rho_2[\int R(\tau+2L/c)\,e^{-i\omega\tau}d\tau+\int R(\tau-2L/c)\,e^{-i\omega\tau}d\tau$$

$$= S_0(\omega)[1+\rho_1\rho_2(e^{i\omega 2L/c}+e^{-i\omega 2L/c}) = S_0(\omega)[1+2\rho_1\rho_2\cos(2\omega L/c)]$$

We see, as before, that the period of the base line ripple is 2L/c. The ratio of maximum to minimum power is $(1+2\,\rho_1\,\rho_2)/(1-2\,\rho_1\,\rho_2)$. Still assuming $|\rho_1\,\rho_2|<<1$, this ratio becomes

$$[Max.pwr/Min.pwr]in\;\;dB = 10\log(1+2|\rho_1\rho_2|)-10\log(1-2|\rho_1\rho_2|)$$

$$= 10\log(e)[\ln(1+2|\rho_1\rho_2|)-\ln(1-2|\rho_1\rho_2|)] \approx 40\log(e)|\rho_1\rho_2|.$$

This is identical to the result of the frequency domain analysis. The former analysis, however, did not require us to assume that the reflection coefficients were small or real or frequency independent.

1.1 Noise resistance of amplifiers
Increased noise from noise mismatch: how much return loss is acceptable in an OMT?

An amplifier, like a bare transistor, is a 2-port device. At any given frequency, its noise characteristics can be described by four parameters. These parameters are commonly chosen to be

$\mathbf{F_{min}}$,the minimum noise figure,

$\mathbf{R_{opt}} + j\mathbf{X_{opt}} = Z_{opt}$ the source impedance that results in $F=F_{min}$,

and $\mathbf{g_n}$, a "noise conductance" that determines the noise figure for non-optimal sources via the equation

$$F = F_{min} + g_n \, |Z_s - Z_{opt}|^2 / R_{opt} \qquad 1)$$

Usually we describe the noise figure in terms of an equivalent "noise temperature"

$$T = T_0 \, [F - 1] = 290 \, [F-1] \qquad 2)$$

Equation 1 then takes the form

$$T = T_{min} + T_{excess} = T_{min} + T_0 \, g_n \, |Z_s - Z_{opt}|^2 / R_{opt} \qquad 3)$$

This can also be written as

$$T = T_{min} + T_{excess} = T_{min} + T_0 \, R_n \, |Y_s - Y_{opt}|^2 / G_{opt} \qquad 4)$$

where $Y_s = 1/Z_s$, $Y_{opt} = 1/Z_{opt} = G_{opt} + jB_{opt}$, and $R_n = g_n R_{opt} / G_{opt}$. this parameter, R_n , is known as the "noise resistance".

The low-noise amplifiers we use are internally *noise matched* to 50 Ohms, i.e. $Z_{opt} = 50 + j0$. A 50-ohm source impedance therefore results in the best (lowest) noise figure. In general, a noise matched amplifier is not simultaneously *impedance matched*; some of the signal incident on the amplifier's input terminal will be reflected. This sometimes causes confusion. How can the amplifier run at optimum sensitivity if it reflects (rejects) some of the available incident power? The answer is that the amplifier's internally-generated noise is to some extent a function of the (external) source impedance since the source impedance is part of the overall circuit in which the amplifier's internal noise sources are embedded. While noise matching results in the loss of some of the input signal through reflection, it simultaneously lowers the internally generated noise and the *signal-to-noise ratio* is maximized.

1.1.1 Noise performance over a given bandwidth

Noise matching, like impedance matching, is frequency dependent. Our amplifiers have their best noise figures at midband, where $Z_{opt} \approx 50 + j0$ and where the antenna/OMT combination presents a 50 source impedance. Away from midband, neither Z_{opt} nor Z_{source} remains equal to 50 ohms. Excess noise is therefore produced according to Equation 3.

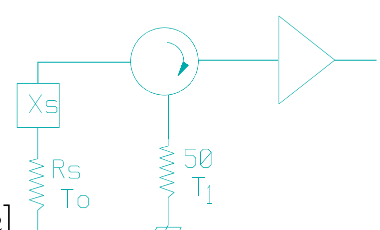
To use this equation, we must know Z_s , Z_{opt} , and T_{min} , and g_n vs. frequency. Of these parameters we can easily measure Z_s , using a network analyzer in place of the amplifier. For most amplifiers, however, we have only a graph of T_{amp} vs. frequency for a flat 50-ohm source. And, while we can reasonably assume that at midband, $Z_{opt} \approx 50 + j0$, we don't know Z_{opt} away from midband. Nor do we know g_n, even at midband. While these various parameters can, in principle, be measured or calculated from a model of the amplifier, the situation is simplified when the amplifier is used with an isolator.

1.1.1.1 Amplifier preceded by an isolator

Amplifiers are often fitted with an isolator at the input terminal. Two reasons for using an isolator are 1. for stability (the amplifier may not be unconditionally stable with respect to the input termination) and 2. to eliminate the signal reflection caused by a noise-matched (but not impedance-matched) amplifier. Reflections are undesirable because, if there is any mismatch between the antenna and the feedline, a mismatch at the amplifier will produce ripples in the overall frequency response. Using an isolator also affects the noise characteristics of the receiving system; we will see below that when an isolator is present, we can predict the noise vs. frequency without knowing g_n or Z_{opt} of the amplifier. All we need to know is the data we have, i.e. T_{amp} vs. frequency for a flat 50-ohm source.

To see how this works, we'll calculate the noise figure of an isolator and then use the "cascade" formula to find the overall noise figure of an amplifier preceded by an isolator. The figure below shows the isolator as a circulator with a 50-ohm termination on the third port. We'll let T_1 represent the physical temperature of this termination.

The power flowing from the circulator to the amplifier is the sum of two terms: the power attributable to R_s and the power attributable to the 50 -ohm termination. Calling this power P_{isol} we can write



$$P_{isol} = \frac{50R_s \cdot 4kT_0}{(50+R_s)^2+X_s^2} + \frac{kT_1\left[(R_s-50)^2+X_s^2\right]}{(50+R_s)^2+X_s^2} \qquad 5)$$

Dividing by the first term (power attributable to the source resistance) gives us the noise figure of the isolator:

$$F_{isol} = 1 + \frac{T_1 |R_x + jX_x - 50|^2}{50 R_s \cdot 4 T_0} \qquad 6)$$

Comparing Equations 6 and 1 we see that Z_{opt} for the isolator is 50 Ohms (no surprise) and that its noise conductance, g_n is given by $T_1 / (4 \cdot 50\ T_0)$.

The combination of the isolator and the amplifier form a cascade whose noise figure is given by the well-known expression

$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} = F_{isol} + \frac{F_{amp} - 1}{G_{isol}} \qquad 7)$$

The gain of the isolator is easily shown to be given by $G_{isol} = 4 \cdot 50 R_s / [(50+R_s)^2 + X_s^2]$. And, since the isolator presents the amplifier with a constant source impedance of 50 ohms, F_{amp} in Equation 7) is the measured test data for the amplifier. Substituting Equation 6 into Equation 7, we find the noise temperature of the isolator/amplifier combination

$$T_{iso/amp} = T_{amp} + \frac{(T_1 + T_{amp})}{4 \cdot 50 R_s} |Z_s - 50|^2 . \qquad 8)$$

We see that the optimum source impedance is a constant 50 ohms over the band of interest and that the noise conductance is given by $(T_1 + T_{amp})/(4 \cdot 50 T_0)$. Since we can measure Z_s as a function of frequency, we can use Equation 8 to calculate the receiver temperature vs. frequency. We can also rewrite Equation 8, expressing $|Z_s - 50|$ and R_s in terms of Γ_s :

$$T_{iso/amp} = T_{amp} + (T_1 + T_{amp}) \frac{|\Gamma_s|^2}{1 - |\Gamma_s|^2} \qquad 9)$$

As a numerical example, suppose that at the band edges, $T_{amp} = 4$ Kelvins and $|\Gamma_s|^2 = .0316$ (15 dB return loss). Suppose also that the physical temperature of the isolator is 16 kelvins. Then $T_{iso/amp}$ = 4 + (16 + 4) .0316/(1-.0316) = 4.065 Kelvins, a rise of only .065 degrees. A 10 dB return loss would raise the temperature by 2.22 Kelvins and a 5 dB return loss would raise the temperature by 9.23 Kelvins.

Klystron Frequency Shift

When the 430 MHz transmitter is used to make Doppler velocity measurements, a correction must be added to the apparent Doppler shift because the output frequency of the klystron amplifier is slightly lower than input frequency (430 MHz). This is a result of the beam velocity modulation used in the klystron and the droop in accelerating voltage that occurs as the capacitor bank is discharged. We can calculate the frequency shift as follows:

Suppose that a concentration of electrons leaves the input cavity at time t_1. Its time-of-arrival at the output cavity will be t_1+D/v, where D is the length of the drift tube (about 2m) and v is the electron velocity (about $c/2$). One period later, at $t_1+1/430E6$, the next concentration of electrons leaves the input cavity. Its time of arrival will be $t_1+1/430E6 + D/(v+\Delta v)$ where Δv is the amount the beam velocity has changed in one period. The error in arrival times (w.r.t. 430MHz) is given by

$$\Delta t_{arrival} = D/(v+\Delta v) - D/v \approx -D\Delta v/v^2.$$

The frequency error is given in terms of this arrival time error by

$$\Delta f/f = -\Delta t_{arrival}/(1/f) = -f\,\Delta t_{arrival}.$$

Therefore we can write

$$\Delta f = -f^2\,\Delta t_{arrival} = f^2\,D\Delta v/v^2.$$

Since $mv^2/2 = qE$, where E is the accelerating voltage, we have $\Delta v/v = \Delta E/2E$ and

$$\Delta f = f^2\,D\Delta E/(2Ev).$$

The voltage change in one cycle, ΔE, is given by $-I(1/f)/C_{PS}$ where C_{PS} is the value of the capacitor bank, 37µF, and I is the beam current. If we assume the output power is 2.5MW, the efficiency is 35%, and the power supply voltage is 100,000, the current is given by $I = 2.5\times 10^6/(0.35\times 100{,}000) = 71A$. Therefore $\Delta E = -71(1/430\times 10^6)/37\times 10^{-6} = -.0045$.

Since the electron total energy is given by $m_0c^2/\sqrt{(1-v^2/c^2)}$ and m_0c^2 = .511Mev, the velocity can be written as

$$v = c\sqrt{1-\left(\frac{511{,}000}{E+511{,}000}\right)^2}$$

When the voltage is 100,000, the velocity is $v = .55c = .55 \times 3 \times 10^8$ *m/s*.
Using these values for ΔE and v, we find that

$$\Delta f = -\,(430 \times 10^6)^2 \times 2 \times .0045/(2 \times 100{,}000 \times .55 \times 3 \times 10^8) = 50.4\ Hz$$

This corresponds to a Doppler velocity of

$$v_{DOPPLER} = c\,\Delta f/(2f) = 50.4(3 \times 10^8)/(2 \times 430 \times 10^6) = 17.6\ m/s$$

Servo Systems (feedback control systems)

Note: This discussion will be in terms of motion control but "motor" can be replaced by the more general term "plant" which can represent a motor, a VCO, or any voltage-controlled "effector".

In the most primitive control system, the motor is simply driven by a current that is proportional to position error. If friction is present (as it always is), the system will not correct a small position error because the small error will produce only a small torque which will not overcome the static friction. This drawback can be solved by adding an op-amp integrator circuit to make the motor drive signal proportional to the time integral of the position error. No matter how small the error is, it will eventually produce a drive signal large enough to overcome the friction. Such a system is known as a *Type 1* control system because it contains one perfect integrator. Type 1 systems, when commanded to a position, will achieve that position with no error. Note that, if no friction is present, the primitive proportional control system needs no op-amp integrator to be a Type 1 system. Without friction, force always produces acceleration which, in turn, results in velocity and change of position. (The frictionless motor is a perfect integrator).

Drive system friction can also be effectively eliminated from the primitive system by adding a tachometer loop (*velocity loop)* around the motor. If this interior loop contains an op-amp integrator, any error will sooner or later produce motion. With the perfect integrator in its forward control path, the system is a Type 1 control system and will reach a commanded position without error.

The VCO in a phase-lock loop is an example of a perfect integrator; the frequency is proportional to the control voltage and the output phase is the integral of the frequency. Any change in control voltage, no matter how small, produces the corresponding change in frequency. There is no dead zone or hysteresis analogous to the friction in a motor drive. While a VCO is indeed a perfect integrator (its control voltage-to-phase transfer function is given by K_i/s, where K_i is a constant), the frictionless motor/tachometer loop merely *contains* a perfect integrator (its transfer function has a single pole at the origin but may have zeros and poles at other locations). Even a frictionless motor has inertia which, in a velocity loop (no other integrator), results in a control voltage-to-position transfer function $K_i/[s(s-s_1)]$, where s_1 is a pole on the negative real axis.

A motor/tachometer velocity loop can be a good approximation to a perfect integrator in a system where the velocities and accelerations are low enough, compared to the motor ratings, to make friction and inertia negligible. In such cases the system design is exactly like that of a PLL and the dynamics will be completely determined by loop filter. In these cases the loop filter is almost always a PI (proportional plus integral) controller and the system dynamics are totally described by two parameters: natural frequency and damping coefficient

Systems with two perfect integrators in the forward loop are known as *Type II* systems. An example is a position servo with an interior velocity loop (one integrator) which is driven by a controller containing an op-amp integrator. A Type II system can track at constant velocity with no error; once the first integrator has built up the necessary velocity command, the input error can go to zero while the first integrator holds that velocity command. Type II systems are appropriate for telescope drive systems since they can provide error-free tracking together with any desired bandwidth (often a low bandwidth is needed for stability). (Note: PLLs, because they are Type 2 loops, will have no phase error if the reference frequency changes. Yet this is not usually needed; as long as the loop reproduces the reference frequency, a phase shift is generally no problem. The reason that Type 2 loops are used in PLLs is to give them the ability to track the reference frequency over large frequency excursions without having more bandwidth than is required by system and stability considerations. The analogous mechanical system is a motor drive servo which may run at a high speed but where adjustments in speed take place in a slow, smooth fashion).

Telescope Drive Systems

The most commonly discussed servo systems are *automatic followers*; they track a "target" whose motion is not predictable. On the other hand, a telescope or machine tool drive system could be called a *position sequencer*; the trajectory is predetermined. When a sequencer is implemented with stepping motors and gear boxes, it can be "open loop" (no feedback). The position that results after a great many steps will be the desired position, except for a small fixed uncertainty given by the variation in step size and backlash in the gear box. Such a system is essentially digital. Note that open loop systems must have enough reserve power to ensure that every step command is obeyed, even when there is considerable force - inertial, frictional, wind, etc.

A follower must obviously use feedback to stay on target. The error signal is used to drive the follower toward the direction of the target. Given the need for feedback, there is no particular advantgage in using digital drive electronics. An analog system can be used; its tendancy to drift is countered by the already-necessary feedback. Correspondingly, a sequencer can be implemented as an analog system with feedback. The desired trajectory can be treated as if it were an unknown target, in which case the sequencer is a standard tracker. But a common technique is to use *velocity feed-forward*, wherein the system is constantly commanded with the correct predetermined velocity. A position error signal is used to increase or decrease the nominal velocity. This allows the system to reach the programmed trajectory from an arbitrary initial position and also allows the system, if the velocity control is not digital, to correct velocity drifts caused by equipmental drifts, and forces from inertia, friction, etc. Note that the feedback loop is really just a correction loop; nominally the velocity feed-forward produces the desired trajectory. And, in so far as the loop is providing only small corrections, the loop gain can be increased for better tracking accuracy. Velocity feed-forward assumes that the system includes a high-performance internal velocity loop. Why not take the idea one step further and use acceleration feed forward. Servo motors really do produce a force proportional to current. If the system has negligible friction, force is proportional to acceleration.

How do we rate the performance of a sequencer? Followers are usually rated by specifying their response to standard inputs: step function, sinewave, realistic target trajectories, etc. A sequencer can be rated by it response to a standard sequence, with and without external disturbances.

8. Polarization

If an electromagnetic wave is propagating in the z-direction, its electric field will, in general, have both a y-component and an x-component. A monochromatic wave can therefore be written as

$$E_x(z,t) = E_1 \cos(\omega t\text{-}kz + \varphi_1) \text{ and } E_y(z,t) = E_2 \cos(\omega t\text{-}kz + \varphi_2) \qquad 1)$$

If either E_1 or E_2 is zero, the wave obviously has *linear* polarization as its electric field is always along either the y-axis or the x-axis. The wave will also be linearly polarized if $\varphi_1 = \varphi_2$ since, for any values of t and z, the **E** field is parallel or anti-parallel to the vector ($E_1\boldsymbol{i} + E_2\boldsymbol{j}$) where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the x and y directions. Circular polarization results when $|E_2| = |E_1|$ and $|\varphi_2\text{-}\varphi_1| = (n + ½)\pi$.

Example of circular polarization

Consider the case where $E_2 = E_1$ and $\varphi_2 = \varphi_1 - \pi/2$. Here we have

$$E_x(z,t) = E_1 \cos(\omega t\text{-}kz + \varphi_1)$$

$$\text{and} \quad E_y(z,t) = E_1 \cos(\omega t\text{-}kz + \varphi_1 - \pi/2) = E_1 \sin(\omega t\text{-}kz + \varphi_1). \quad 2)$$

The magnitude of **E** is independent of z and t since

$$[E_x(z,t)]^2 + [E_y(z,t)]^2 = E_1^2 [\cos^2 (\omega t\text{-}kz + \varphi_1) + \sin^2 (\omega t\text{-}kz + \varphi_1)] = E_1^2 \quad 3)$$

In any xy plane (z = constant), this wave's electric field vector rotates from the positive x axis towards the positive y axis, etc. If our coordinate axes have the usual handedness, i.e. if **i** x **j = k** , then this wave has *right-hand circular polarization*. (If we look into an approaching wave and see the E-vector rotating counter-clockwise, the polarization is right-hand circular as defined by electrical engineers and radio astronomers).

The four real parameters E_1, φ_1, E_2, and φ_2 describe any monochromatic wave as a superposition of two linearly polarized waves. Linear and circular polarizations are special cases; in general, $|E_1|$ ... $|E_2|$ and $|\varphi_1 - \varphi_2|$ ... $(n + ½)\pi$ and the polarization is elliptical. It is common practice to use two complex amplitudes ("phasors") E_1 and E_2 in order to express the wave as

$$E_x = Re(\mathsf{E}_1\, e^{j[\omega t - kz]}) \text{ and } E_y = Re(\mathsf{E}_2\, e^{j[\omega t - kz]}). \qquad 4)$$

The complex amplitudes are simply $\mathsf{E}_1 = E_1\, e^{j\varphi}{}_1$ and $\mathsf{E}_2 = E_2\, e^{j\varphi}{}_2$. 5)

Phasor representations are also useful to describe a wave of arbitrary polarization in terms of two circularly polarized waves. Let us write

$$E_x = Re\, \mathsf{E}(t,z)$$
$$E_x = Im\, \mathsf{E}(t,z)$$
$$where = \mathsf{E}(t,z) = \mathsf{E}_R e^{j[\omega t - kz]} + \mathsf{E}_L\, e^{-j[\omega t - kz]} \qquad 6)$$

Here, E_x and E_y are, respectively, the real and imaginary parts of a phasor, E, which is the sum of two counter-rotating phasors, E_R and E_L . If E_R is zero, the polarization of the wave is left-hand circular. If E_L is zero, the polarization of the wave is right-hand circular. In general, polarization is elliptical and both E_R and E_L are non-zero. To express E_R and E_L in terms of E_1 and E_2, we first rewrite Equation 6) as

$$E_x = Re([\mathsf{E}_R + \mathsf{E}_L^*]e^{j[\omega t - kz]}) \text{ and } E_y = Re([-j\, \mathsf{E}_R + j\, \mathsf{E}_L^*]e^{j[\omega t - kz]}). \qquad 6a)$$

Comparing 6a) with 4) we see that

$$E_R + E_L^* = E_1 \quad \text{and} \quad -jE_R + jE_L^* = E_2 . \qquad 7)$$

Solving 7) for E_R and E_L , we find

$$E_R \;=\; ½(E_1 + j\,E_2) \quad \text{and} \quad E_L \;=\; ½(\,E_1 - jE_2)^*. \qquad 8)$$

The total power is given by

$$P_{tot} \;=\; |E_x|^2 + |E_y|^2 \;=\; |E_1|^2 + |E_2|^2 \;=\; 2|E_R|^2 + 2|E_L|^2. \qquad 9)$$

If we are given E_1 and E_2, the complex amplitudes of the linear components, we can use Equations 8 and 9 to express the ratio of RHC power to LHC power:

$$P_R\,/\,P_L \;=\; |E_R|^2 / |\,E_L|^2 \;=\; |\,E_1 + j\,E_2|^2 /\; |\,E_1 - j\,E_2|^2 \qquad 10)$$

In the general case of elliptical polarization, we often want to specify the axial ratio and the orientation of the ellipse. The axial ratio is the ratio of the minimum-to-maximum magnitudes of the sum of the left hand and right hand phasors. (Visualize the ellipse formed by the sum of the counter-rotating vectors).

$$\mathit{Axial\ Ratio} \;=\; [|E_R| - |E_L|] \,/\, [|E_R| + |E_L|]$$

$$=\; [\;|E_1 - jE_2| - |E_1 + jE_2|\;] \;\;/\;\; [\;|E_1 - jE_2| + |E_1 + jE_2|\;]. \qquad 11)$$

To find the orientation of the polarization ellipse, we rewrite 6) as

$$E_x = Re(E_R e^{j\theta} + E_L e^{-j\theta}) \quad \mathit{and} \quad E_y = Im(E_R e^{j\theta} + E_L e^{-j\theta}). \qquad 12)$$

where $\theta = \omega t\text{-}kz$. To find the major axis of the ellipse, we find θ_m , the value of θ for which $|E|^2 = E_x^{\,2} + E_y^{\,2}$ is a maximum. Expressing $|E|^2$ we have

$$|E|^2 = | E_R e^{j\theta} + E_L e^{-j\theta} |^2 = | E_R |^2 + |E_L|^2 + 2|E_L| |E_R|(\cos [2\theta+\varphi_R-\varphi_L]) \qquad 13)$$

This expression is obviously a maximum when $2\theta_m + \varphi_R - \varphi_L = 2\pi$ or $\theta_m = (2\pi-\varphi_R+\varphi_L)/2$.
If we denote by Φ the angle from the positive x-axis to the major axis of the ellipse, we have

$$\Phi = \arg (E_R e^{j\theta_m} + E_L e^{-j\theta_m}) \qquad 14)$$

Expressing this in terms of the x and y representation,

$$\Phi = \arg \{ (E_1 + j E_2)\, e^{j (2\pi - \varphi_R + \varphi_L)/2} + (E_1 - jE_2)^* e^{-j (2\pi - \varphi_R + \varphi_L)/2} \} \qquad 15)$$

where $\varphi_R = \arg E_R = \arg (E_1 + j E_2)$

and $\varphi_L = \arg E_L = \arg (E_1 - j E_2)^*$.

We have seen that four parameters (two amplitudes and two phases or two complex amplitudes) completely describe a monochromatic wave in time and space. Usually we are not interested in the absolute phase of the wave. Three parameters are sufficient to specify the power and polarization. Two parameters are sufficient to specify the polarization.

Example of elliptical polarization

Consider the arbitrary case where $E_2 = 0.87$, $E_1 = 0.44$, $\varphi_2 = -135$ deg, and $\varphi_1 = - 94$ deg.
Using the analysis described above, we find that the polarization ellipse has an axial radio of 0.286 and that the major axis of the ellipse is at an angle of 67.136 degrees. The ratio of LHC power to RHC power is 3.241.
(See attached Mathcad sheet).

Suppose the electric field is expressed as follows in terms of its x and y components:

$$E1 := .44 \qquad \phi 1 := -94 \cdot deg$$

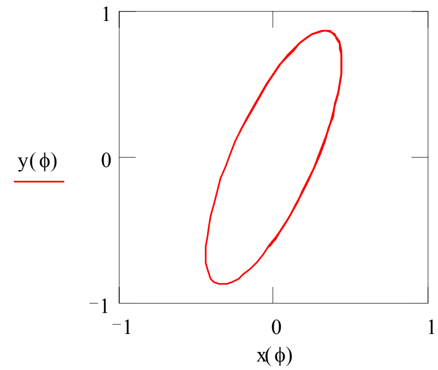
$$E2 := .87 \qquad \phi 2 := -135 \cdot deg$$

$$\phi := 0, .1 .. 10$$

$$x(\phi) := E1 \cdot \cos(\phi + \phi 1)$$

$$y(\phi) := E2 \cdot \cos(\phi + \phi 2)$$

This field is shown in the plot at the right.



Expressing the field in terms of x and y phasors, we have

$$EE1 := E1 \cdot e^{j \cdot \phi 1}$$

$$EE2 := E2 \cdot e^{j \cdot \phi 2}$$

The same field can be expressed in terms of L and R phasors (Left and Right circular polarization):

$$EEL := .5 \cdot \left(\overline{(EE1 - j \cdot EE2)}\right)$$

$$EER := .5 \cdot (EE1 + j \cdot EE2)$$

For this example, the ratio of Right Circular power to Left Circular power is:

$$PwrLtoR := \frac{(|EER|)^2}{(|EEL|)^2} \qquad \text{or} \qquad \left(\left|\frac{EE1 + j \cdot EE2}{\overline{(EE1 - j \cdot EE2)}}\right|\right)^2 = 3.241$$

$$PwrLtoR = 3.241$$

And the axial ratio is given by

$$Axialratio = \frac{||EER| - |EEL||}{|EER| + |EEL|} \qquad \text{or} \qquad \frac{||(EE1 + j \cdot EE2)| - |(EE1 - j \cdot EE2)||}{|(EE1 - j \cdot EE2)| + |(EE1 + j \cdot EE2)|} = 0.286$$

$$Axialratio = 0.286$$

The phase (in terms of circular components) at which the electric vector reaches a maximum is given by

$$\theta m := \frac{1}{2} \cdot (2 \cdot \pi - \arg(EER) + \arg(EEL))$$

The orientation of the ellipse w.r.t. the x axis is given by

$$\left(\frac{180}{\pi} \cdot \arg\left(EER \cdot e^{j \cdot \theta m} + EEL \cdot e^{-j \cdot \theta m}\right)\right) = 67.136$$

or, in terms of the x and y phasors,

$$\frac{180}{\pi} \cdot \arg\left[(EE1 + j \cdot EE2) \cdot e^{\frac{j}{2} \cdot \left(2 \cdot \pi + \arg\left(\frac{\overline{(EE1 - j \cdot EE2)}}{EE1 + j \cdot EE2}\right)\right)} + \left(\overline{(EE1 - j \cdot EE2)}\right) \cdot e^{\frac{-j}{2} \cdot \left(2 \cdot \pi + \arg\left(\frac{\overline{(EE1 - j \cdot EE2)}}{EE1 + j \cdot EE2}\right)\right)}\right] = 67.136$$

Unpolarized radiation

For the monochromatic radiation described above, the power and the polarization have no time variation. Such radiation is produced by a single cw generator connected to one or more antennas or by multiple cw generators, all with the same frequency, feeding any arbitrary arrangement of antennas. Any cw source is polarized.

But suppose now that we are observing radiation that is produced by two independent noise generators, one connected to an *X* antenna and the other to a *Y* antenna. The x and y signals, as seen through the bandpass filters of a dual-channel radio astronomy receiver, will each be sine waves with varying amplitudes and phases. Over a time interval that is short compared to the reciprocal of the bandwidth, there is an apparent polarization ellipse. But long time averages will show no correlation between the x and y components. Such a signal is said to be unpolarized. Note, however, that a source consisting of one or more noise generators connected to antennas of the same polarization would be a polarized source. The nature of the polarization, (axial ratio and orientation of the polarization ellipse) would be determined by the antenna(s). Likewise, a collection of elementary radiators having random phases but with identical polarization, determined, for example, by an external magnetic field, will be a polarized source.

In general radio astronomy sources are partially polarized; a fraction of the power has random polarization while the rest of the power has a definite polarization ellipse. Any source is therefore completely described by four parameters: the unpolarized power, the polarized power, the axial ratio of the polarization ellipse, and the orientation of the ellipse.

Polarization Measurements

A common technique is to observe the signal with X and Y receivers or with R and L receivers, measuring four quantities: the averages of the squares of the voltages from each receiver, the average of product of the voltages from the two receivers, and the average of the product of voltages when one voltage has been delayed a quarter cycle with respect to the other (delaying a phasor by a quarter cycle effectively multiplies the phasor by $e^{-j\pi/2} = -j$). These products can be produced from the two IF signals by using by four multipliers (mixers). The output voltage from each multiplier, after filtering away the components at twice the IF frequency, is the product of the amplitudes of the two input phasors times the cosine of the angle between them. If the two inputs of a multiplier are connected to the same input signal, the output is just the square of the amplitude of that signal.

We can express the x and y phasors as E_1 and E_2, in terms of their unpolarized and polarized components: $E_1 = E_{1u} + E_{1p}$ and $E_2 = E_{2u} + E_{2p}$. The four multiplier outputs are will then be

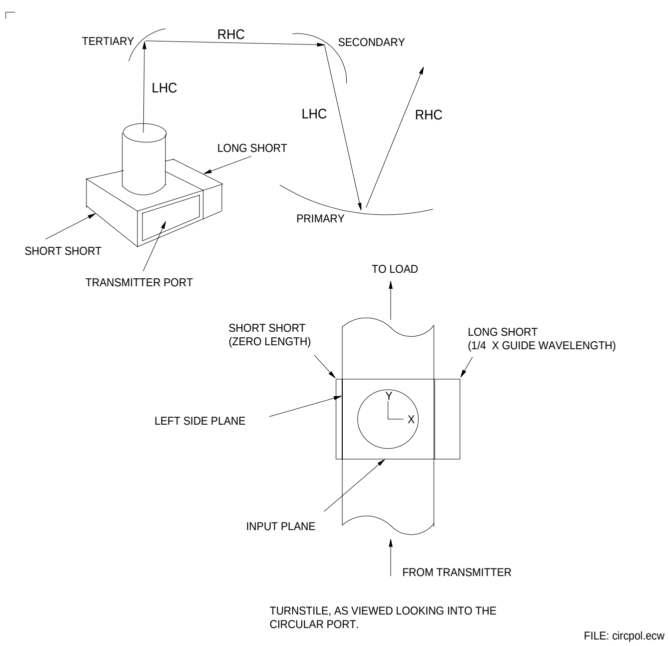
$$= <(E_{1u} + E_{1p})(E_{1u}^* + E_{1p}^*)> = <(E_{1u} E_{1u}^* > + < E_{1p} E_{1p}^*)>$$
$$<E_2 E_2^* > = <(E_{2u} + E_{2p})(E_{2u}^* + E_{2p}^*)> = <(E_{2u} E_{2u}^* > + < E_2 E_2^*)>$$
$$<E_1(-jE_2 > = <(E_{2u} + E_{2p})(E_{2u}^* + E_{2p}^*)> = <(E_{2u} E_{2u}^* > + < E_2 E_2^*)>$$

$$<E_1 E_1^* > = <(E_{1u} + E_{1p})(E_{1u}^* + E_{1p}^*)> = <(E_{1u} E_{1u}^* > + < E_{1p} E_{1p}^*)>$$

$$<E_2 E_2^* > = <(E_{2u} + E_{2p})(E_{2u}^* + E_{2p}^*)> = <(E_{2u} E_{2u}^* > + < E_2 E_2^*)>$$
$$<E_1(-jE_2 > = <(E_{2u} + E_{2p})(E_{2u}^* + E_{2p}^*)> = <(E_{2u} E_{2u}^* > + < E_2 E_2^*)>$$

Sense of Polarization from turnstile junction orthomode transducer

Summary: The turnstile, as pictured below, produces left-circular polarization. After reflections from the tertiary, secondary, and primary mirrors, the radiated polarization is right-circular.



We consider the polarization of the wave coming out of the turnstile. We will take the phase of the y-component to be the phase at the input plane. In our S-band turnstile, the phase path from the input plane to the left side plane is almost exactly 270 degrees, as shown by lab measurements and by HFSS simulation. The wave making this left turn is then shifted another 180 degrees when it reflects from the short short. The total phase of the x-component is therefore 270+180 = 90 degrees. Expressing the x and y components, we have

$$E_x = \mathrm{Re}\,(e^{j\omega t-\pi/2}) = \cos(\omega t-\pi/2) = \sin(\omega t)$$

$$E_y \quad = \mathrm{Re}(e^{j\omega t}) \; = \; \cos(\omega t)$$

The E vector, as seen when the wave is *approaching* us, thus rotates in the *clockwise* direction. (As t increases from zero, E_x goes from zero to a positive number while E_y begins to decrease). By definition (IRE, IEEE) this is *left-circular* polarization. The tertiary, secondary, and primary reflectors each reverse the polarization so the telescope finally transmits *right-circular* polarization.