To: File
From: Jon Hagen
Date: July 5, 2002
Subject: Calculating static torque of a Kollmorgen motor from the dc currents in the three phases

Summary: The torque constant for the B606A "brushless dc" motor is specified as $k_T = 1.651$ ft lbs/ RMS Amp. When the motor is rotating at a constant speed, the current on each of the three input wires is a sine wave. These three sine wave currents have equal magnitudes and the same frequency (proportional to the motor speed), but have relative phases of 0, 2π/3, and 4π/3. If the RMS value of the current (which can be measured in any one of the wires if the motor is turning) is 10 amps, the motor is supplying a torque of $10\ k_T = 16.51$ ft lbs. When the shaft is held motionless, the torque is again given by $T = k_T\ I_{RMS}$, but, in this case, the equivalent dc RMS current is calculated as $I_{RMS} = [(I_1^2 + I_2^2 + I_3^2)/3]^{1/2}$.

Discussion: Suppose the shaft is held motionless. The currents in each wire will be dc (zero frequency ac) and the three current values will be different. The amplifier, using the resolver data, makes the three values proportional to cosθ, cos(θ+2π/3) and cos(θ+4π/3), where θ is the shaft angle (or an integral multiple of the shaft angle). Let the three currents be written as

$$I_1 = I_{pk} \cos\theta,\ I_2 = I_{pk}, \cos(\theta+2\pi/3),\ \text{and}\ I_3 = I_{pk}\cos(\theta+4\pi/3). \qquad 1a,b,c)$$

The permanent magnet rotor makes the torque contributions from the three currents proportional to cos θ, cos(θ+2π/3) and cos(θ+4π/3), so the total torque is given by

$$T = \alpha\ (I_1 \cos\theta + I_2 \cos(\theta+2\pi/3) + I_3 \cos(\theta+4\pi/3)) \quad \text{or}$$

$$T = \alpha\ I_{pk} \{\ (\cos\theta)^2 + (\cos(\theta+2\pi/3))^2 + (\cos(\theta+4\pi/3))^2\ \} = 1.5\ \alpha\ I_{pk} \qquad 2)$$

where α is a constant proportional to k_T. Note that the term in the curly brackets is identically equal to 1.5 for any value of θ; this type of motor has no torque ripple as the shaft turns.

From the definition of k_T, and the fact that RMS current is just peak current divided by √, we have

$$T = 1.5\ \alpha\ I_{pk} = k_T\ I_{RMS} = k_T\ I_{pk} /\surd \qquad 3)$$

so α must be given by

$$\alpha = k_T /(1.5\ \surd) \qquad 4)$$

and

$$T = k_T I_{pk} /\sqrt{\ } \qquad 5)$$

Let us form the root of the sum of the average squares of I1, I2, and I3. This RMS value is

$$I_{RMS} = [(I_1^2 + I_2^2 + I_3^2)/3]^{1/2} . \qquad 6)$$

Substituting from Equations 1a,b, and c, we find $I_{RMS} = 1.5 \; I_{pk} /\sqrt{\ }$ or

$$I_{pk} = \sqrt{\ } I_{RMS} \; /1.5 \qquad 7)$$

Substituting Equation 7 into Equation 5, we have

$$T = k_T I_{pk} /\sqrt{\ } \;=\; k_T \sqrt{\ } I_{RMS} \; /(1.5 \sqrt{\ }) \;=\; k_T I_{RMS} \qquad 8)$$

which is the desired result. As an experimental check, a B606A motor was attached to the torque wrench and the drive was increased to produce an indicated torque of 20 ft lbs. The magnitudes of the three currents were measured using the clip-on Hall effect ammeter as 2.1A, 13.8A, and −15.7A. Using value from the motor's specification sheet, $k_T = 1.651$ ft lbs/A we have

ft lbs

which agrees well with the indicated 20 ft lbs. The algebraic sum of the measured currents is

which indicates a slight measurement error, since this sum should be identically zero.

Notes from 6-24-02
More testing in digital lab.

6V control voltage produced 40 ft lbs and a torque monitor voltage of 4.24V
When cranked to saturation, the system went up to 55 ft lbs and a monitor voltage of 6.6 V.
It then folded back to 40 ft lbs, again with a monitor voltage of 4.24V.

Torque/control voltage is 40/6 = **6.67 ft lbs/control volt**

How much torque is needed per motor?

Wt of Gregorian = 160,000 lbs

160,000/8 sin za x Rp = T in ftlbs x transmission ratio (transmision ratio = 190.07)

where Rp is radius of pinion (half the pitch diamter) (1/2 (.2667m) = 5.25"=.4375')

So T in ftlbs = 160,000/8 x sin za x .4375 / 190.07 = 46.03 x sin za ft lbs.

T at 20 deg = 46.03 x sin(20 deg) = **15.74 ft lbs/motor**

Sum of Torques at 20 deg = 8 x av torque/motor = **126 ft lbs**

The I monitor port gives an approximation to the desired rms of the current. Probably it gives max | Ii| of the three phases. This varies between 1.4 and 1.6 of the true value 1.5, i.e. up to 6.7%too high and 6.7% too low. The Kollmorgen BD4 Amplifier Manual says:
Pin 18 I monitor
Torque/monitor voltage = 40/4.24 = 9.43 ftlb/monitor volt or 55/6.6 = 8.33 ftlb / monitor volts (average is **8.88 ft lb / monitor volt**).

Compare with specs from instruction manual:
Manual says 8 monitor volts for peak current (40 amps) or **5 amps per monitor volt**
which would be 8 volts for 40 x 1.651 = 66 ftlbs. or 66/8 = **8.255 ft lb per monitor volt**.

Non-linear relation between Torque and Monitor voltage.
The Kollmorgen manual states “There is a direct relationship between the signal appearing at this output and actual motor current. A dc voltmenter placed between pin 18 and common can be used to estimate the constant load levels placed on the motor. The current scale factor is 8V = Reak RMS current rating of the BDS4 (3k Ohm oput impedance). This output is for reference only. Its accuracy decreases as current decreases: - 4% at peak current +/- 9 % at continuous current, +/- 12 % at 1/2 continous current. “ (Moreover, the voltage on pin 18 is always positive - it doesn’t indicate the sign of the torque. Couldn’t they have done better than this?!).

Setting the Amplifier Scale Gain

The amplifier (dome amplifiers) scale factor is to be adjusted for **6.67 ft lbs/control volt.** Adjust the scale pot as follows: Align the motor shaft key with the base of the pointer in the torque tester. Set the control voltage to -5.5Volts. Adjust the *scale* pot until the *Isense* voltage reading is 4.0 Volts. The torque should be 36.7 ft. lbs for this control voltage. (The pot may well be at its fully clockwise position). Maximum continuous torque, +/-33.4 ft lbs corresponds to a control voltage of +/- 5V. Maximum current (2 seconds before fold back) corresponds to a control voltage of +/- 10V (full scale).

Interpreting the Torque Monitor Voltage

Lab test data, Vcontrol vs. Vmonitor, suggest the following relation:

|Torque in ft. lbs| = 13.3(Vmonitor)$^{.73}$ for Vmonitor <4
|Torque in ft. lbs| = 6.47 + 7.6 Vmonitor for Vmonitor <4

This is derived from the data shown below, where the solid line is 2(Vmonitor)$^{.73}$ for Vmonitor >4 and .97 +1.14 Vmonitor for Vmonitor>4. These data were taken using the torque wrench set-up at various shaft angles.

If a little error is acceptable at the top of the scale, the curve can be represented entirely by
|Torque in ft. lbs| = 13.3(Vmonitor)$^{.73}$ for Vmonitor <4, as shown below.