

1.0 PROJECTED FATIGUE LIFE REMAINING

Using the data provided by NAIC staff, a damage ratio was calculated to determine how much fatigue life had already been used up of the original channel members in U2-U2L3. Assuming a 50 kip residual load in each of the original channels, a new stress range was calculated based on the reinforced condition. Using the same yearly average, the projected remaining cycles and remaining years of the inner channels was calculated.

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BY WA DATE 5/5/11 PROJECT ARECIBO SHEET No. 1 OF 1

CKD. BY _____ DATE _____ SUBJECT FATIGUE

N_{LIFE} 207680

CYCLES/YR. 4684

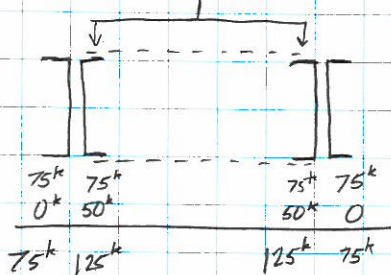
CYCLES SINCE 1995 (15 YRS) 70260

DAMAGE RATIO 0.338

RESIDUAL 100^k LOAD IN U2-U2L3 (50^k IN EACH CHANNEL)
ORIGINAL I's

$$P_{max} = 400^k - 100^k \text{ (RESIDUAL DL)} = 300^k$$

$$P_{min} = 50^k$$



$$A_{NET} = 4.475 \text{ IN}^2$$

$$f_{y,max} = 28 \text{ ksi}$$

$$f_{y,min} = 3.5 \text{ ksi}$$

$$F_{SR} = 24.5 \text{ ksi}$$

PREDICTED LIFE REMAINING

$$N_{LIFE} = \frac{120 \times 10^3}{(24.5)^3} = 815,987 \text{ CYCLES}$$

$$\text{CYCLES REMAINING} = (1 - 0.338) \times 815,987 = \underline{\underline{540,183 \text{ CYCLES}}}$$

$$\text{YEARS REMAINING} = \frac{540,183}{4684} = \underline{\underline{115 \text{ YEARS}}}$$

2.0 DATA PROVIDED BY NAIC

The average cycles per year used in the calculations is the summation of all cycles greater than 15,000 kip feet. These numbers are shown in the chart in the middle of the page.

Summary:

Number of telescope cycles using Amplitude of unbalanced moment change:

Number of telescope cycles
integrated 2004.5 thru 2010

Amplitude KipFeet	corner 12 totCyles	corner 4 totCycles	corner 8 TotCycles
> 5000	48665	53061	52568
> 10000	29838	28155	27620
> 15000	15469	15536	14985
> 2000	8320	8773	8384

- The above table shows the accumulated cycles for the 6.5 years.

Median Cycles/year

Amplitude KipFeet	Median Yearly cycles
> 5,000	9378
> 10,000	5418
> 15,000	2980
>20,000	1704

- The median cycles/year has excluded the 2004,2007, and 2010 data

Unbalanced moments for typical motions:

- Some of the motions used on the telescope:
 - World day az swings:
 - dome=15, ch=15 = 9500 Kipfeet.
 - dome=15, ch=0 = 13500 kipFeet.
 - Galfacts za nodding
 - az=0, dome 2 to 19.6 deg
 - max unbalanced moment: 18500 KipFeet
 - min unbalance moment: - 9000 KipFeet
 - Usually 3 cycles/hour
 - swinging from source set to source rising
 - will have a moment of +/- 19500 kipfeet *cos(azRise,set - corneraz)

- it can take 1.5 to 2.5 hours to track a source rise to set.
- The tiedown tensions were not used at all in this computation.

3.0 FATIGUE LIFE CALCULATIONS

Calculations for the expected fatigue life for U2-U2L3, U1-L2, and U3-L4 based on the different loads from the original design, the Gregorian upgrade in 1995, 2010 analysis loads, and 2010 analysis loads after reinforcement. From the 13th Edition AISC Steel Construction Manual come the stress category constants and threshold fatigue stress ranges. The number of estimated lifetime cycles at the various stress ranges for both net and gross sections was calculated.

AISC 13th Edition Appendix 3.3

$C_{fA} := 250 \cdot 10^8$	Stress Category A Constant
$C_{fB} := 120 \cdot 10^8$	Stress Category B Constant
$F_{THA} := 24 \cdot \text{ksi}$	Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category A
$F_{THB} := 16 \cdot \text{ksi}$	Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category B
$A_{C8} := 5.51 \cdot \text{in}^2$	Gross area of C8x18.75
$n := 2$	Number of bolts
$d_h := 1 \cdot \text{in} + \frac{1}{16} \cdot \text{in} = 1.063 \cdot \text{in}$	Bolt hole diameter
$t_w := 0.487 \cdot \text{in}$	C8x18.75 web thickness
$A_{net} := A_{C8} - n \cdot d_h \cdot t_w = 4.475 \cdot \text{in}^2$	Net area of C8x18.75

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.1 Base metal, except non-coated weathering steel, with rolled or clean surface. Flame-cut edges with surface roughness values of 1000 micro inches or less, but without reentrant corners.

Potential crack initiation point is away from all welds or structural connections.

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.3 Member with drilled or reamed holes. Member with reentrant corners at copes, cuts, block-outs or other geometrical discontinuities made to requirements of Appendix 3.5, except weld access holes.

Potential crack initiation point is at any external edge or at hole perimeter.

4-C8x18.75 WITH 2010 ANALYSIS LOADS

Stress Category = B

$$P_{\max} := \frac{397 \cdot \text{kip}}{4} = 99.25 \text{ kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{51 \cdot \text{kip}}{4} = 12.75 \text{ kip}$$

Minimum load in member U2-U2L3

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8}} = 18.01 \cdot \text{ksi}$$

Maximum stress at gross section

$$f_{\min} := \frac{P_{\min}}{A_{C8}} = 2.314 \cdot \text{ksi}$$

Minimum stress at gross section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 15.7 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 3101614$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

$$F_{SR} < F_{TH}$$

Therefore, member has infinite life cycles. However, inner channels have had previous cycles used up and member may not have complete infinite life.

$$F_{SR} = 15.7 \cdot \text{ksi} < F_{THB} = 16 \cdot \text{ksi}$$

Net Section (Design Parameter 1.3)

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}}} = 22.18 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}}} = 2.849 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 19.33 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 1661684$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

2-C8x18.75 WITH 2010 ANALYSIS LOADS

Stress Category = B

$$P_{\max} := \frac{397 \cdot \text{kip}}{2} = 198.5 \text{ kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{51 \cdot \text{kip}}{2} = 25.5 \text{ kip}$$

Minimum load in member U2-U2L3

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8}} = 36.03 \cdot \text{ksi}$$

Maximum stress at gross section

$$f_{\min} := \frac{P_{\min}}{A_{C8}} = 4.628 \cdot \text{ksi}$$

Minimum stress at gross section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 31.4 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 387702$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

Net Section (Design Parameter 1.3)

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}}} = 44.36 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}}} = 5.698 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 38.66 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 207710$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

2-C8x18.75 WITH 1992 REINFORCING DESIGN LOADS

Stress Category = B

$$P_{\max} := \frac{316 \cdot \text{kip}}{2} = 158 \cdot \text{kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{84 \cdot \text{kip}}{2} = 42 \cdot \text{kip}$$

Minimum load in member U2-U2L3

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8}} = 28.68 \cdot \text{ksi}$$

Maximum stress at gross section

$$f_{\min} := \frac{P_{\min}}{A_{C8}} = 7.623 \cdot \text{ksi}$$

Minimum stress at gross section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from drawings

$$F_{SR} = 21.05 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 1286062$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).**Net Section (Design Parameter 1.3)**

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}}} = 35.31 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}}} = 9.385 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 25.92 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 689006$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

2-C8x18.75 WITH ORIGINAL 1960 DESIGN LOADS

Stress Category = A

$$P_{\max} := \frac{291 \cdot \text{kip}}{2} = 145.5 \text{ kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{108 \cdot \text{kip}}{2} = 54 \text{ kip}$$

Minimum load in member U2-U2L3

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8}} = 26.41 \cdot \text{ksi}$$

Maximum stress at gross section

$$f_{\min} := \frac{P_{\min}}{A_{C8}} = 9.8 \cdot \text{ksi}$$

Minimum stress at gross section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from drawings

$$F_{SR} = 16.61 \cdot \text{ksi}$$

$$N := \frac{C_{fA}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 5459232$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1). $F_{SR} < F_{TH}$ Therefore, member has infinite life cycles.

$$F_{SR} = 16.606 \cdot \text{ksi} < F_{THA} = 24 \cdot \text{ksi}$$

AISC 13th Edition Appendix 3.3

$C_{fA} := 250 \cdot 10^8$	Stress Category A Constant
$C_{fB} := 120 \cdot 10^8$	Stress Category B Constant
$F_{THA} := 24 \cdot \text{ksi}$	Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category A
$F_{THB} := 16 \cdot \text{ksi}$	Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category B
$A_{C8} := 5.51 \cdot \text{in}^2$	Gross area of C8x18.75
$n := 2$	Number of bolts
$d_h := 1 \cdot \text{in} + \frac{1}{16} \cdot \text{in} = 1.063 \cdot \text{in}$	Bolt hole diameter
$t_w := 0.487 \cdot \text{in}$	C8x18.75 web thickness
$A_{net} := A_{C8} - n \cdot d_h \cdot t_w = 4.475 \cdot \text{in}^2$	Net area of C8x18.75

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.1 Base metal, except non-coated weathering steel, with rolled or clean surface. Flame-cut edges with surface roughness values of 1000 micro inches or less, but without reentrant corners.

Potential crack initiation point is away from all welds or structural connections.

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.3 Member with drilled or reamed holes. Member with reentrant corners at copes, cuts, block-outs or other geometrical discontinuities made to requirements of Appendix 3.5, except weld access holes.

Potential crack initiation point is at any external edge or at hole perimeter.

2-C8x18.75 WITH 2010 ANALYSIS LOADS

Stress Category = B

$$P_{\max} := \frac{324 \cdot \text{kip}}{2} = 162 \cdot \text{kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{32 \cdot \text{kip}}{2} = 16 \cdot \text{kip}$$

Minimum load in member U2-U2L3

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8}} = 29.4 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{C8}} = 2.904 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 26.5 \cdot \text{ksi}$$

$$N := \frac{C_{fA}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 1343805$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).**Net Section (Design Parameter 1.3)**

$$A_{\text{net.total}} := A_{\text{net}} + (7 \cdot \text{in} \cdot 0.5 \cdot \text{in} - 2 \cdot 0.5 \cdot \text{in} \cdot d_h) = 6.913 \text{ in}^2$$

$$f_{\max} := \frac{P_{\max}}{A_{\text{net.total}}} = 23.44 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net.total}}} = 2.315 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 21.12 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 1273655$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

2-C8x18.75 WITH 2010 ANALYSIS LOADS - REINFORCED
WITH 4-L3x2x5/16

Stress Category = B

$$P_{\max} := \frac{324 \cdot \text{kip}}{2} = 162 \cdot \text{kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{32 \cdot \text{kip}}{2} = 16 \cdot \text{kip}$$

Minimum load in member U2-U2L3

Net Section (Design Parameter 1.3)

$$A_L := 1.46 \cdot \text{in}^2$$

$$f_{\max} := \frac{P_{\max}}{A_{C8} + \left(2A_L - d_h \cdot \frac{5}{16} \cdot \text{in}\right)} = 20.01 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{C8} + \left(2 \cdot A_L - d_h \cdot \frac{5}{16} \cdot \text{in}\right)} = 1.976 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{SR} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{SR} = 18.03 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{SR}}{\text{ksi}}\right)^3} = 2047628$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

Net Section (Design Parameter 1.3)

Maximum stress at net section

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}} + 2\left(A_L - d_h \cdot \frac{5}{16} \cdot \text{in}\right) + (7 \cdot \text{in} \cdot 0.5 \cdot \text{in} - 2 \cdot 0.5 \cdot \text{in} \cdot d_h)} = 17.67 \cdot \text{ksi}$$

Minimum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}} + 2\left(A_L - d_h \cdot \frac{5}{16} \cdot \text{in}\right) + (7 \cdot \text{in} \cdot 0.5 \cdot \text{in} - 2 \cdot 0.5 \cdot \text{in} \cdot d_h)} = 1.745 \cdot \text{ksi}$$

$$F_{\text{SR}} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{\text{SR}} = 15.92 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{\text{SR}}}{\text{ksi}}\right)^3} = 2971847$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

Gross Section (Design Parameter 1.1)

$$f_{\max} := \frac{P_{\max}}{A_{C8} + 2 \cdot A_L} = 19.22 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{C8} + 2 \cdot A_L} = 1.898 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{\text{SR}} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{\text{SR}} = 17.32 \cdot \text{ksi}$$

$$N := \frac{C_{fA}}{\left(\frac{F_{\text{SR}}}{\text{ksi}}\right)^3} = 4812427$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

AISC 13th Edition Appendix 3.3

$$C_{fA} := 250 \cdot 10^8$$

Stress Category A Constant

$$C_{fB} := 120 \cdot 10^8$$

Stress Category B Constant

$$F_{THA} := 24 \cdot \text{ksi}$$

Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category A

$$F_{THB} := 16 \cdot \text{ksi}$$

Threshold fatigue stress range, maximum stress range for indefinite design life (Table A-3.1) for stress category B

$$A_{C10} := 5.87 \cdot \text{in}^2$$

Gross area of C10x20

$$n := 2$$

Number of bolts

$$d_h := 1 \cdot \text{in} + \frac{1}{16} \cdot \text{in} = 1.063 \cdot \text{in}$$

Bolt hole diameter

$$t_w := 0.379 \cdot \text{in}$$

C10x20 web thickness

$$A_{net} := A_{C10} - n \cdot d_h \cdot t_w = 5.065 \cdot \text{in}^2$$

Net area of C10x20

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.1 Base metal, except non-coated weathering steel, with rolled or clean surface. Flame-cut edges with surface roughness values of 1000 micro inches or less, but without reentrant corners.

Potential crack initiation point is away from all welds or structural connections.

Design Parameter:

Section 1 - Plain Material Away from Any Welding

- 1.3 Member with drilled or reamed holes. Member with reentrant corners at copes, cuts, block-outs or other geometrical discontinuities made to requirements of Appendix 3.5, except weld access holes.

Potential crack initiation point is at any external edge or at hole perimeter.

2-C10x20 WITH 2010 ANALYSIS LOADS

Stress Category = B

$$P_{\max} := \frac{307 \cdot \text{kip}}{2} = 153.5 \cdot \text{kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{16 \cdot \text{kip}}{2} = 8 \cdot \text{kip}$$

Minimum load in member U2-U2L3

Net Section (Design Parameter 1.3)

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}}} = 30.31 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}}} = 1.58 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{\text{SR}} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{\text{SR}} = 28.73 \cdot \text{ksi}$$

$$N := \frac{C_{\text{fB}}}{\left(\frac{F_{\text{SR}}}{\text{ksi}}\right)^3} = 506097$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

2-C10x20 WITH 2010 ANALYSIS LOADS - REINFORCED

Stress Category = B

$$P_{\max} := \frac{307 \cdot \text{kip}}{2} = 153.5 \cdot \text{kip}$$

Maximum load in member U2-U2L3

$$P_{\min} := \frac{16 \cdot \text{kip}}{2} = 8 \cdot \text{kip}$$

Minimum load in member U2-U2L3

Net Section (Design Parameter 1.3)

$$A_p := \frac{5}{8} \cdot \text{in} \cdot 8 \cdot \text{in} - n \cdot d_h \cdot \frac{5}{8} \cdot \text{in} = 3.672 \cdot \text{in}^2$$

$$f_{\max} := \frac{P_{\max}}{A_{\text{net}} + A_p} = 17.57 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{\text{net}} + A_p} = 0.916 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{\text{SR}} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{\text{SR}} = 16.65 \cdot \text{ksi}$$

$$N := \frac{C_{fB}}{\left(\frac{F_{\text{SR}}}{\text{ksi}}\right)^3} = 2597794$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).**Gross Section (Design Parameter 1.1)**

$$f_{\max} := \frac{P_{\max}}{A_{C10}} = 26.15 \cdot \text{ksi}$$

Maximum stress at net section

$$f_{\min} := \frac{P_{\min}}{A_{C10}} = 1.363 \cdot \text{ksi}$$

Minimum stress at net section

$$F_{\text{SR}} := f_{\max} - f_{\min}$$

Stress range from analysis

$$F_{\text{SR}} = 24.79 \cdot \text{ksi}$$

$$N := \frac{C_{fA}}{\left(\frac{F_{\text{SR}}}{\text{ksi}}\right)^3} = 1641592$$

Number of stress range fluctuations in design life
Equation derived from Equation (A-3-1).

4.0 MINER'S RULE

Information regarding the basis of Miner's Rule and how it is applied.

13.1 Calculating Lifetime according Miners rule

The Palmgren-Miner Linear-cumulative-fatigue-damage-theory (Miner's Rule) is used to calculate the resultant pitting or bending fatigue lives for gears that are subjected to loads which are not of constant magnitude but vary over a wide range. According to Miner's Rule, failure occurs when:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_i}{N_i} = 1$$

where:

- n_i = number of cycles at the i^{th} stress level.
- N_i = number of cycles to failure corresponding to the i^{th} stress level.
- n_i/N_i = damage ratio at the i^{th} stress level.

Instead of load cycles we can also use lifetimes:

$$\frac{l_1}{L_1} + \frac{l_2}{L_2} + \dots + \frac{l_i}{L_i} = 1$$

where:

- l_i = time at a the i^{th} stress level.
- L_i = permissible lifetime at the i^{th} stress level.
- l_i/L_i = damage ratio at the i^{th} stress level.

Assuming the fraction of time at each stress level is known rather than the actual number of cycles or times, then:

$$\begin{aligned} l_1 &= \alpha_1 \cdot L \\ l_2 &= \alpha_2 \cdot L \\ l_i &= \alpha_i \cdot L \end{aligned}$$

where:

- α_i = fraction of time at the i^{th} stress level.
- L = Resultant number of cycles to failure under the applied load spectrum.

Defining the time ratio as:

$$\alpha_i = l_i/L = n_i/N$$

Miner's Rule may be rewritten as:

$$\alpha_1 \frac{L}{L_1} + \alpha_2 \frac{L}{L_2} + \dots + \alpha_i \frac{L}{L_i} = 1$$

Which may be solved for the resultant life:

$$L = \frac{1}{\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \dots + \frac{\alpha_i}{L_i}}$$

The load spectrum is defined by the time ratio, α_i , and the load ratio, β_i and additionally a speed ratio ω_i is needed for the calculation of the permissible lifetimes L_i .

where:

$$\begin{aligned}\beta_i &= \text{instantaneous load/baseline load} \\ \omega_i &= \text{instantaneous speed/nominal load}\end{aligned}$$

The baseline load is entered with the Load Data input screen by specifying the transmitted horsepower and speed of the pinion. The load spectrum is entered on the page Lifetime: