Uniform Aperture Approximation

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I. Far Fields from a Circular Aperture with Uniform Illumination

The normalized far fields produced by a circular aperture of radius a with uniform illumination is given by

$$E_\theta(\theta,\phi) = 2\,\frac{J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta}\,\cos\phi \tag{1}$$

$$E_\phi(\theta,\phi) = 2\,\cos\theta\,\frac{J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta}\,\sin\phi \tag{2}$$

$$\begin{aligned}|\mathbf{E}(\theta,\phi)|^{\mathbf{2}} &= E_\theta(\theta,\phi)^2 + E_\phi(\theta,\phi)^2 \\ &= \left(2\,\frac{J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta}\right)^2 \left(\cos^2\phi + \cos^2\theta\,\sin^2\phi\right)\end{aligned} \tag{3}$$

For $\phi = 0$ we get:

$$E_\theta(\theta,\phi) = 2\,\frac{J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta} \tag{4}$$

$$E_\phi(\theta,\phi) = 0 \tag{5}$$

$$|\mathbf{E}(\theta,\phi)|^{\mathbf{2}} = \left(2\,\frac{J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta}\right)^2 \tag{6}$$

The HPBW is obtain by solving

$$\left(\frac{2\,J_1(k\,a\,\sin\theta)}{k\,a\,\sin\theta}\right)^2 = 0.5 \tag{7}$$

The solution is:

$$\frac{2\,\pi}{\lambda}\,a\,\sin\theta_{HPBW} = 1.61634 \tag{8}$$

Solving for a in wavelengths,

$$\frac{a}{\lambda} = \frac{0.257249}{\sin\theta_{HPBW}} \tag{9}$$

II. Arecibo Aperture Efficiency Calculation

Arecibo's elliptical aperture of 237m $\times$ 207 m corresponding to a physical area of 38530.85 m^2. One first order approximation is to assume a circular aperture, and use Equation 9 to obtain a value of aperture efficiency based on HPBW measurements.

A better approximation is to use the far field intensity distribution due to an elliptical aperture [Kathuria, 1983]

$$|\mathbf{E}(\theta,\phi)|^{\mathbf{2}} = \left(\frac{2\, J_1(k\sqrt{a^2\,\sin^2\theta\,\cos^2\phi + b^2\,\sin^2\theta\,\sin^2\phi})}{k\sqrt{a^2\,\sin^2\theta\,\cos^2\phi + b^2\,\sin^2\theta\,\sin^2\phi}} \right)^2 \quad (10)$$

where, a and b are the major and minor semi-axis of the elliptical aperture respectively. Therefore, by measuring θ_{HPBW} at $\phi = 0$ and $\phi = 90^o$ we obtain,

$$\frac{a}{\lambda} = \frac{0.257249}{\sin\theta_{HPBW}|_{\phi=0}} \quad (11)$$

$$\frac{b}{\lambda} = \frac{0.257249}{\sin\theta_{HPBW}|_{\phi=90^o}} \quad (12)$$

REFERENCES

[Kathuria, 1983] Y. P. Kathuria, " Far-Field Radiation Patterns of Elliptical Apertures and its Annulli ", *IEEE Transactions on Antennas and Propagation*, Vol AP-31, No. 2, March 1983, pp. 360-364.